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Abstract

Understanding the duration of widowhood is essential for individuals and effective widow support policies. However, widowhood duration (WD) remains an understudied topic. In this article, we provide a quantitative estimation of the impact of three primary determinants of expected WD at age 60 within a unified framework: *(i)* the degree of overlap between male and female mortality distributions; *(ii)* the age gap between spouses; and *(iii)* the dependence of spousal mortality. Using French life tables from 1962 to 2070 and simulations based on the Gompertz law and a bivariate Gaussian copula, we assess each determinant's relative influence. Our findings show that ignoring spousal mortalities dependence overestimates WD by three years, while disregarding the age gap underestimates it by one year. In France, in 2020, expected WD for females at age 60 is 10.4 years and 5.8 years for males. Despite converging gender life expectancies, our projections suggest that WD will remain high until 2070: 9.2 years for females and 6.2 years for males. Notably, we identify a negative gradient of WD along the standard of living distribution.

Key-words: widowhood duration, differential mortality, life tables, Gompertz mortality law, bivariate Gaussian copula.

JEL classification: J11; J12; J14; J18.

Introduction

The death of a spouse is one of the most devastating shocks an individual can face, due not only to the emotional impact of losing a family member but also to the substantial financial risk and far-reaching economic implications for the survivor.[1] Knowing the durations individuals spend in this vulnerable situation is of paramount importance for both policymakers and the individuals themselves. First, widowhood duration (WD) holds crucial significance as a key parameter for policymakers, as many countries have implemented public survivor insurance schemes aimed at mitigating the decline in household standard of living following the death of a spouse (OECD, 2019). The duration of survivor benefit provisions has a critical impact on the sustainability of the pension schemes. Moreover, WD may vary among individuals due to factors such as gender or socioeconomic background, thus potentially increasing the risk of poverty among elderly individuals living alone (Munnell et al., 2020). This risk may be exacerbated if a correlation exists between survivors' life expectancy and their income, as suggested by previous research (Chetty et al., 2016). Policymakers are motivated to address this correlation in order to minimize welfare concerns. Second, comprehending the expected lifespan of survivors after the death of their spouse plays a vital role in enabling individuals to make informed decisions throughout their life-cycle (De Nardi et al., 2021). It has been observed that both females and males tend to underestimate their likelihood of becoming a widow or widower, primarily because they overestimate the number of years they expect to live alongside their spouse (Holden and Kuo, 1996).[2] Such underestimations may lead to inadequate savings, whether through public or private means, due to an inappropriate time horizon.This is one contributing factor to the economic challenges faced by widows (Munnell et al., 2020).

Until recently, WD has received limited attention in the literature, with noteworthy exceptions being the recent work by Compton and Pollak (2021) and pioneering studies by Goldman and Lord (1983) and Myers (1959). These studies, based on U.S. life tables, have examined the historical evolution of WD, identifying variations based on educational

[1]Indeed, at spousal death, at least three mechanisms directly modify household resources: *(i)* the survivor no longer benefits from the deceased spouse's income; *(ii)* the survivor loses economies of scale; and *(iii)* depending on certain criteria, the survivor may receive survivor's benefits.

[2]This tendency to underestimate remaining life expectancy as a widow is partially attributed to a broader underestimation of one's own life expectancy, as previously noted in the literature (Kutlu-Koc and Kalwij, 2017; Dormont et al., 2018).

attainment and race. While they consider factors such as the overlap between female and male mortality distributions and the average age gap between spouses in the population, they do not delve into the dispersion of age gaps or the dependence of spousal mortality. Similarly, The literature on couples' joint survival and survivors' life expectancy has attracted considerable interest in the fields of risk and insurance, particularly for annuity computation purposes (Frees et al., 1996; Denuit et al., 2001; Youn and Shemyakin, 2001), as well as in the welfare literature aimed at measuring the effect of spousal death (Strand, 2005; Leroux and Ponthiere, 2013; Ponthiere, 2016).[3] These studies do consider the dependence of spousal mortality. However, these two strands of the literature do not focus specifically on WD as a condition of whether one is either a widow or a widower. In other words, they explicitly address WD at the macro level but overlook both WD within the widowed population and disparities across socioeconomic characteristics. Although the literature on WD has generally failed to fully quantify its determinants, recent interest among demographers has arisen regarding the probability of males outliving females, as in the work of (Bergeron-Boucher et al., 2022), which itself built upon a broader research framework measuring the inequality of lifespans between two populations (Vaupel et al., 2021). Both papers highlight that even though, on average, women tend to live longer than men, a notable proportion of men have a significant chance of living longer than women. Despite these valuable insights into the overlap between male and female lifespan distributions, a critical aspect remains unresolved: the duration of outsurvival, which is central to our understanding of WD.

This article estimates the impact of three primary determinants of WD within a unified framework: *(i)* the mortality distributions of both females and males, including their degree of overlap; *(ii)* the age gap between spouses; and *(iii)* the dependence of spousal mortality. Through simulations based on Gompertz law and a bivariate Gaussian copula, we quantitatively assess the relative influence of each determinant. Our findings show that ignoring spousal mortality dependence overestimates WD by three years, while disregarding the age gap underestimates it by one year. In France, in 2020, expected WD for females at age 60 was 10.4 years and 5.8 years for males. Despite converging gender life expectancies, our projections suggest that WD will remain high until 2070: 9.2 years

[3]The United States Social Security Administration also provides an interactive software called *Longevity Visualizer* that enables visual-analysis of individual and joint survival probabilities of two people (Alleva, 2016).

for females and 6.2 years for males. Notably, we identify a negative gradient of WD along the standard of living distribution.

This paper contributes to several strands of the literature. First, we apply the same methodology as Myers (1959), Goldman and Lord (1983) and Compton and Pollak (2021) to French mortality tables, documenting both the level of WD and its evolution in France over the 1962–2020 period. Prior research had focused primarily on the United States. Second, we expand on the outsurvival literature by enhancing the outsurvival indicator, initially defined by Vaupel et al. (2021), namely by introducing a temporal dimension to measure WD. Third, using Gompertz law simulations and a bivariate Gaussian copula, we further contribute to these strands by estimating the influence of WD determinants within a unified framework, thus facilitating a comparison of their relative impacts. This includes quantifying, for the first time, the significance of both the dispersion of the age gap between spouses and the dependence of spousal mortality when measuring WD. Fourth, we project our analysis until 2070, going beyond the historical evolution of WD as described in prior research (Compton and Pollak, 2021). We not only document that WD has been notably substantial over the past decades but also highlight that it is projected to decrease, albeit to a lesser extent than what might be inferred from the narrowing gap in average life expectancies between females and males. Finally, we modestly contribute to documenting the correlation between income and life expectancy (Chetty et al., 2016). We show that WD decreases linearly with standard of living and is more dispersed in the lower part of the income distribution, increasing the risk of very long widowhood and, in turn, poverty.

Measuring Widowhood Duration

Background

Consider a heterosexual couple consisting of same-age spouses whose respective ages at death are independent. Let ω represent the maximum lifespan, and x denote the age at which expected widowhood duration is computed. The variables for deaths, survivals, and life expectancies at age u are denoted as $d(u)$, $s(u)$ and $e(u)$, respectively. Let index f refers to females and m to males. The life expectancy of the wife at age x is calculated as:

$$LE_f(x) = \int_x^\omega \frac{s_f(u).du}{s_f(x)} \tag{1}$$

We assume negligible mortality before age x and set $s_f(x) = s_m(x) = 1$. The probability that a woman survives at age $u \geq x$ is $s_f(u)$, and the probability that her spouse dies at this age is $d_m(u)$. Therefore, she has a $s_f(u).d_m(u)$ probability of becoming a widow at age u. If this event occurs, she has $e_f(u)$ years left to live. The expected WD at age x is then derived from the product of these terms, summed over all possible values of u.

$$\begin{aligned} WD_f(x) &= \int_x^\omega d_m(u).s_f(u).e_f(u).du & (2)\\ &= \int_x^\omega d_m(u).s_f(u).\left[\int_u^\omega \frac{s_f(v)}{s_f(u)}dv\right].du & (3)\end{aligned}$$

We next define $k_f(u) = \int_u^\omega s_f(v)dv$, which verifies $k_f'(u) = -s_f(u)$.
The previous expression can thus be written as:

$$WD_f(x) = \int_x^\omega d_m(u).k_f(u).du = -\int_x^\omega s_m'(u).k_f(u).du, \tag{4}$$

which we integrate by parts:

$$[-s_m(u).k_f(u)]_x^\omega + \int_x^\omega s_m(u).k_f'(u).du = -s_m(\omega).k_f(\omega) + s_m(x).k_f(x) - \int_x^\omega s_m(u).s_f(u).du.$$

The first term of this expression is null, as $s_m(\omega)$ is null.
Since $k_f(x) = \int_x^\omega s_f(u).du$ and $s_m(x) = 1$, we have:

$$WD_f(x) = \int_x^\omega s_f(u).[1 - s_m(u)].du = LE_f(x) - JS(x), \tag{5}$$

where $JS(x)$ is the joint survival at age x. In other words, we measure WD as the female life expectancy minus the joint survival of the spouses. This duration is computed for all women, whether they outlive their husband or not.[4] When referring to WD in everyday language, we typically mean WD conditional on experiencing spousal death, that is, the ratio between unconditional widowhood duration (UWD) and the probability

[4]In other words, we include in the calculation the null WD of women who died before their husbands.

that a woman outlives her husband (*proba*). This conditional WD is then equal to:

$$WD_f^C(x) = \frac{\int_x^\omega e_f(u).s_f(u).d_m(u).du}{\int_x^\omega s_f(u).d_m(u).du} = \frac{UWD_f(x)}{proba_f(x)} = \frac{LE_f(x) - JS(x)}{1 - \Phi(x)}, \quad (6)$$

where Φ is the outsurvival statistic introduced by Vaupel et al. (2021).

$$\Phi(x) = \int_x^\omega s_m(u).d_f(u).du, \quad (7)$$

Φ measures the probability that an individual from a low life expectancy group will live longer than an individual with a high life expectancy. In this context, males constitute the low life expectancy group and females the high life expectancy group. $\Phi(x)$ then measures the probability that males outlive females at age x and equals one minus the probability that females outlive males, namely widowhood probability.

In the following sections, WD refers to widowhood duration conditional on being a widow(er) and we use UWD when referring to unconditional widowhood duration.

Determinants of Widowhood Duration

Mortality Distributions Overlap

When considering mortality distributions, it's important to acknowledge that there is an overlap between the mortality patterns of different groups. If all females were to outlive all males, there would be no male widowhood, and WD would simply be the difference between the average life expectancies of females and males. This common misconception underscores that WD is influenced by variations around the average age at death and the degree of overlap between the mortality distributions of females and males. For a more detailed framework explaining the impact of mortality overlap on WD, see online Appendix A.1, where we show that calculating WD as a simple difference between individual life expectancies of spouses can result in negative counts when the order of deaths is reversed. However, understanding the extent to which overlap affects WD cannot be divorced from empirical investigation. The degree of overlap depends on the characteristics of mortality distributions, which have undergone substantial changes in the past and will continue to do so in the future. These changes, including processes such as mortality distribution compression and rightward shifts, are well-documented (Bergeron-Boucher et al., 2015).

Age Gap between Spouses

On average, women tend to marry older men (Goldman and Lord, 1983; Drefahl, 2010). For a given woman, marrying an older man increases her probability of widowhood at a young age compared to marrying a same-age partner, as older men have higher death rates than their younger counterparts. This, in turn, may also increase the duration of widowhood because the younger a woman is at her spouse's death, the longer her expected years as a survivor. This insight underscores the role of the age gap between spouses in widowhood. To provide a formal framework for understanding UWD, we extend the numerator of equation (6) to accommodate bivariate distributions of the age of the wife and the age of the husband:

$$UWD_f(x) = \int_{u=x}^{\omega} \int_{z=a}^{b} \int_{y=A}^{B} e_f(y+u).s_f(y+u).d_m(z+u).dy.dz.du, \tag{8}$$

where a and b are the youngest and oldest ages of husbands and A and B are the youngest and oldest ages of wives. In the simplified scenario where the age gap between spouses is constant across the population, we provide more details in online Appendix A.2 showing that we introduce an upward bias into the WD measure if we assume that a fixed age difference of Δ adds Δ years of widowhood to all women. This assumption overlooks the fact that this difference will not prevent some women from passing away before their husbands, while others may survive less than Δ years after their husbands' deaths. Despite the average age gap remaining relatively stable over time, variations within the population may still play a significant role. In light of this, empirical simulations are warranted to assess the influence of the age gap and its dispersion on widowhood.

Dependence of Spousal Mortality

Spouses often exhibit assortative mating patterns according to their socioeconomic characteristics (Gonalons-Pons and Schwartz, 2017). However, age at death is not randomly distributed in the population, with higher socio-economic status (SES) individuals typically experiencing later mortality than those with lower SES. This implies a correlation in ages at death within couples. The shared risks and standards of living during their common life further amplify this phenomenon. Additionally, the well-documented 'broken heart effect' which leads to increased mortality among widowers (Parkes et al., 1969;

Elwert and Christakis, 2008; Shor et al., 2012), underscores that the assumption of independence of spousal mortality is not valid. The field of risk insurance has extensively explored the dependence of spousal mortality as a means to measure joint survival and determine survivors' life expectancy, particularly for annuity calculations (Frees et al., 1996; Denuit et al., 2001; Youn and Shemyakin, 2001). We can empirically examine the role of the dependence of spousal mortality within the context of WD measurement by using bivariate copula functions commonly used in such calculations.

Data and Methods

French Life Tables

We utilize the life tables from the French Institute for Statistical and Economic Studies (INSEE) for both females and males. These tables offer age-specific mortality distributions spanning from 1962 to 2020, and they include projected mortality tables from 2021 to 2070, specifically for metropolitan France (Algava and Blanpain, 2021). Additionally, we incorporate the 2016 life tables categorized by standard of living as computed by (Blanpain, 2018) using French fiscal data. Blanpain (2018) determines the standard of living vingtiles for the entire population, pooling together both females and males, while also accounting for differences across various age groups.

Computation using Life Tables

We calculate WD by adapting Equation (6) to discrete-time, employing mortality tables for randomly paired couples. WD measures how long a surviving spouse can expect to live after the death of their current spouse, without considering remarriage. This approach aligns with existing literature, particularly the work of Compton and Pollak (2021). Our choice of the 60-year-old threshold is grounded in two additional justifications. First, the credibility of assuming no divorce before widowhood and no remarriage after widowhood increases as divorce rates dramatically decrease after age 40 (Prioux and Barbieri, 2012; Kennedy and Ruggles, 2014); and, second, remarriage rates significantly decline with age (Smith et al., 1991; Wu et al., 2014).

In our primary WD definition, we also assume a two-year age difference between spouses ($\Delta = 2$), which corresponds to the observed median (mean is 2.5 years), for

French married couples (Papon, 2019). More precisely, the numerator is the cumulated sum from 60 to ω, calculated as the product of three different terms: *(1)* the number of men who were two years older than women and died at the given age; *(2)* the number of female survivors at this age; and *(3)* the female life expectancy at this age. The denominator is computed as the sum from age 60 to ω of the product of *(a)* the number of male deaths at the considered age, and *(b)* the number of female survivors at this age.

$$WD^f = \frac{\sum_{a=60}^{\omega} d^m_{a+2} . \frac{S^f_a + S^f_{a+1}}{2} . \frac{e^f_a + e^f_{a+1}}{2}}{\sum_{a=60}^{\omega} d^m_{a+2} . \frac{S^f_a + S^f_{a+1}}{2}} \tag{9}$$

For males, the equation is similar, with the replacement of f by m. It is important to note that, in these WD calculations for randomly paired couples, we have so far assumed no correlation in the mortality of spouses, although we will do so later in the paper.

Simulations based on Gompertz Distributions

To illustrate how changes in the shape of mortality distributions lead to shifts in the overlap between male and female distributions, subsequently affecting probability of widowhood and, ultimately, WD, we utilize Gompertz mortality distributions calibrated to INSEE's life tables. The Gompertz law is particularly well-suited to our purposes for two reasons. First, this probability law is widely used for modeling human mortality (Olshansky and Carnes, 1997). Online Appendix B.1 illustrates the quality of the fit to our data.[5] Second, Gompertz law parameters can be easily estimated and used to compare survival curves of different populations over time. For all ages $x \geq 0$ the density of a Gompertz distribution is:

$$f_g(x, a, b) = a.exp\left[bx - \frac{a}{b}(e^{bx} - 1)\right]. \tag{10}$$

If we set $u = \frac{1}{b} ln(\frac{b}{a})$ and $c = \frac{1}{b}$, the Gompertz density can be rewritten as a location-scale mortality density (Basellini et al., 2019):

$$f(x, c, u) = \frac{1}{c} exp\left[\frac{x-u}{c} - exp\left(\frac{x-u}{c}\right) + exp\left(-\frac{u}{c}\right)\right], \tag{11}$$

[5]To allow the replicability of our results, we have included all parameter estimates used in this paper in Table B.1

where $u \in \mathbb{R}$ is the location parameter, and $c > 0$ is the scale parameter of the distribution. An increase in the location parameter u shifts the mortality distribution to the right, corresponding to the well-documented increase in the modal age at death over the past decades. The scale parameter c represents the variability in the distribution. Changing the scale parameter affects how far the probability distribution stretches out. A decrease in c may be considered as a compression of age at death distribution around the mode.

We further introduce an age-gap between spouses. To do so, we randomly pair 100,000 females and 100,000 males into heterosexual couples whose respective ages at death were drawn from Gompertz laws, and we assign them an age gap drawn from a Normal distribution. Individual WD is equal to the difference between the spouse's respective date of death plus the age gap between spouses.[6]

Finally, indrawing from research in the actuarial literature, we introduce the dependence of spousal mortality by employing a bivariate Gaussian copula (Kaishev et al., 2007). Let T_1 and T_2 be two random variables (female and male mortality distributions), and $F_1(t_1) = P(T_1 \leq t_1)$ and $F_2(t_2) = P(T_2 \leq t_2)$ be their distribution functions. The joint distribution is $F(t_1, t_2) = P(T_1 \leq t_1, T_2 \leq t_2)$. According to Sklar (1959), there must exist a copula C such that:

$$F(t_1, t_2) = C(F_1(t_1), F_2(t_2)), \tag{12}$$

and if F_1 and F_2 are continuous, C is unique. In practice, we use a bivariate Gaussian copula:

$$\begin{aligned} C(u_1, u_2) &= P(U_1 \leq u_1, U_2 \leq u_2) && (13) \\ &= P(T_1 \leq \Phi^{-1}(u_1), T_2 \leq \Phi^{-1}(u_2) \cup T_2 \leq t_2) && (14) \\ &= \Phi_2\left(\Phi^{-1}(u_1), \Phi^{-1}(u_2); \rho\right), && (15) \end{aligned}$$

where Φ is the cumulative distribution function (CDF) of a standard normal distribution and $\Phi_2(\rho)$ is the joint CDF of (T_1,T_2). We utilize a Gaussian copula because it allows for a straightforward representation of the rank correlation between the two considered

[6] We calculate UWD for the population under consideration by averaging individual WD values; any negative WD values (corresponding to cases where the wife dies before the husband) are adjusted to zero. WD conditional on becoming a widow is computed as the average of strictly positive individual WD values.

distributions (Klaassen and Wellner, 1997). This enables us to easily determine the sequence in which the members of the couple pass away. The Gaussian copula has a parameter ρ controlling the strength of dependence. More precisely, we first draw an age at death in a Gompertz distribution for 100,000 females and independently do the same for 100,000 males. We then rank the two populations according to their age at death. We then use the simulated marginal distributions $F(t_1, \infty)$ and $F(\infty, t_2)$ of the Gaussian copula to determine how ranks are correlated and pair the 100,000 females and 100,000 males according to this correlation pattern. Finally, we compute WD as the difference between the matched spouse's respective dates of death and their corresponding age gap (drawn from the Normal distribution).

Results

Trends in Widowhood Duration

In France, in 2020, the expected WD at age 60 for women stood at 13 years (Figure 1) assuming a two-year age gap with their spouse. The UWD was lower since it was calculated for all women, regardless of whether they outlived their husbands or not (see Background section). In 2020, the expected UWD at age 60 was 9 years. The probability of widowhood stood at 70%, and is consistent with Bergeron-Boucher et al. (2022). From 1962 to 2020, WD, UWD and the probability of widowhood displayed relative stability. Males exhibit a lower probability of widowhood but its duration is significant. In 2020, the expected male WD at age 60 reached 9 years. Notably, this duration has increased slightly by one year since 1962. Online Appendix B.2 show that our results hold when we compute WD at age 50 instead of age 60 or when we switch from a period to a cohort analysis.[7]

The Role of Widowhood Duration Determinants

We now employ simulations to illustrate and quantify the role of WD determinants. Our approach unfolds progressively as we first examine the overlap of mortalities, then

[7] The expected WD at age 50 reaches 14.7 years for females and 10.4 years for males (Appendix Figure B.2a). Appendix Figure B.2b displays the evolution of the expected WD at age 60 across birth cohorts (with a two-year age gap between spouses). Females born in 1902 have an expected WD of 13.6 years, while their male counterparts have a WD of 8.9 years.

introduce an age gap between spouses, and, finally, incorporate dependence of spousal mortality.

We begin with a scenario featuring no age gap or dependence of mortality between spouses. Figure 2 provides a graphical assessment of the role played by overlapping mortalities in WD values. Panel (a) displays female and male mortality distributions and the degree of the overlap. Panel (b) depicts the probability of widowhood in accordance with the Equation (6) denominator, and it represents the difference between the area under the male mortality curve and the area under the male mortality curve multiplied by the female survival curve. Finally, panel (c) illustrates the UWD, corresponding to the area between the joint survival and female life expectancy curves, as per the Equation (6) numerator. To evaluate the impact of mortality overlap, we performed four simulations, each corresponding to the four figures in each panel (a) to (c) of Figure 2. In these simulations, we progressively adjusted the scale and location parameters for the female distribution to match their actual values. From left to right, the first figure depicts the scenario in which women have the same mortality pattern as men (with identical location and scale parameters). In the second and third figures, we assume that only the location parameter (in the second figure) and only the scale parameter (in the third figure) of the Gompertz law for women is the same as that for men. Finally, in the fourth figure, women and men have their respective actual mortality patterns. If females and males shared the same mortality patterns, then using male mortality rates in 2020 as a reference (first figure, panel (a)) would result in perfect overlap. In this scenario, females would be equally likely to die first or become widows, resulting in a 50% probability of widowhood (first figure, panel (b)). Given the survival curve characteristics in 2020 (location parameter of 86.8 years and scale parameter of 9.6), the UWD would be 5.4 years (first figure, panel (c)). Consequently, the expected WD at age 60, conditional on being a widow, would be 10.9 years. Empirically, females have a higher life expectancy that is characterized by a right-shifted mortality distribution compared to men (with a greater location parameter in the Gompertz law) and a more compressed distribution (lower scale parameter in the Gompertz law). The compression of the female distribution does not alter the probability of widowhood (second figure, panel (b)) but slightly reduces the UWD to 5.2 years (second figure, panel (c)). On the other hand, shifting the female distribution to the right (third figure, panel (a)) increases both the probability of widowhood (63%, third figure, panel (b)) and the UWD (8.1 years), resulting in a total WD of 12.9 years (third figure, panel (c)). Finally, considering both the current scale and location parameters of the female mortality distribution, the probability of widowhood reaches 64% (fourth figure, panel (b)), UWD reaches 7.9 years (fourth figure, panel (c)), and the total amounts to a WD of 12.4 years.[8] In summary, it appears that only females' higher modal age at death contributes to the increase in WD compared to a scenario in which females and males share the same mortality patterns in 2020.

Figure 3 shows the sensitivity of WD both to the age gap between spouses and to the dependence of spousal mortality. In panel (a), we represent WD for different values of the age gap, drawn from a Normal distribution with mean ranging from 0 to 4 and a standard deviation of 5, aligning with the findings of Bouchet-Valat (2019). In 2020, the expected WD at age 60 for women is 13.4 years, assuming spouses have an average age gap of 2.5 years and a standard deviation of 5 for age gaps in the population.[9] WD shows a positive correlation with the average age gap in the population. Given the characteristics of female and male mortalities in 2020, adding one year to the age gap between spouses has a marginal effect of contributing 0.36 years to WD. [10]

In panel (b), we vary the degree of dependence between females and males mortalities (ρ parameter of the Gaussian copula) from 0 (independence) to 1 (perfect correlation). When ρ=0, WD equals 13.4 years, aligning with our previous results. If we assume a perfect correlation (ρ=1), WD reduces to the simple difference between average female life expectancy and average male life expectancy, as this corresponds to the scenario in which every woman dies after her husband.[11] The stronger the dependence of spousal mortality, the lower the WD (Figure 3b). For $\rho = 0.6$ (Kaishev et al., 2007), WD is 10.4 years.

Figure 4 summarizes the influence of these different determinants. In 2020, the expected WD at age 60 is 10.4 years. If we disregard the dependence of spousal mortality,

[8]Note that the different methodology results in a slight difference in WD compared to the previous section (Figure 1 shows WD in 2020 to be 12.5 years, not 12.4 years). Online Appendix C provides a detailed description of the reasons for these minor discrepancies.

[9]Online Appendix B.3 provides an intermediate step in which we exclude age gap dispersion. WD would then be only 13 years.

[10]Neglecting the dispersion in the age gap distribution would have led to the conclusion that the age gap plays a lesser role in WD: the marginal effect of adding one year to the age gap is equal to 0.25, consistent with life table computations.

[11]See online Appendix B.3, panel (c) for WD without any age gap between spouses. For ρ=1, WD equals 4.6 years, representing the difference between female life expectancy at age 60 (27.8 years in 2020) and male life expectancy at age 60 (23.2 years in 2020).

WD would increase by 3 years, reaching 13.4 years. Further, if we ignore the presence of an age gap between spouses, WD would decrease by 1 year to 12.4 years. When considering that the dispersion of ages at death for females is equal to that for males, WD would increase by 0.5 years to 12.9 years. Finally, if we additionally assume that females' mortality pattern matches that of males[12], WD would be 10.9 years, representing a 2-year decrease.

Projections and Heterogeneity Results

The increase in life expectancy throughout the 20th century can be attributed to both mortality compression and the shift of mortality to older age groups (Bergeron-Boucher et al., 2015; de Beer and Janssen, 2016).[13] These crucial demographic factors are expected to continue evolving into the future. A study conducted by (Basellini and Camarda, 2019), focusing on Denmark, France, Japan, and Sweden, predicts a rise in the modal age at death and a reduction in lifespan variability. These changes may, in turn, impact WD. In this section, we project WD from 2020 to 2070 using INSEE projected life tables. Figure 5 illustrates the projected evolution of WD over this time span. Female WD is forecasted to decline from 10.4 years in 2020 to 9.2 years in 2070. Subsequently, the overlap between female and male mortalities increases as a result from two key factors: the compression of mortality rates in both females (with the scale parameter expected to decrease from 8.1 to 7.6) and males (with the scale parameter expected to decrease from 9.6 to 7.7); and the shift toward older ages in both genders (with the location parameters expected to increase from 91.7 to 95.8 for females and from 86.8 to 93.7 for males).[14] The probability of widowhood is projected to decrease by 7pp (–9%) from 75% to 68% between 2020 and 2070, albeit to a lesser extent than the UWD, which is expected to decrease by 20% from 7.9 to 6.3 years during the same period. These findings remain robust across various simulation scenarios.[15] Male WD is projected to experience a slight

[12]The same modal and dispersion of ages at death.

[13]Mortality compression was observed during the first half of the twentieth century in Sweden Bergeron-Boucher et al. (2015) and in the Netherlands, de Beer and Janssen (2016). In the second half of the century, this dynamic was primarily replaced by the shifting of the mortality schedule, while lifespan variability remained relatively constant.

[14]Online appendix B.4 provides graphical illustrations for the evolution of the overlap of spousal mortalities and its role in WD.

[15]In addition to running simulations using the extreme life expectancy scenarios provided by INSEE, we conducted simulations in which *(1°)* the age gap between spouses and *(2°)* the dependence of spousal mortality vary by ±10% around the central scenario (online Appendix B.5). With the exception of the high life expectancy scenario, in which WD is forecasted to increase very slightly (up to 11 years), the

increase over this period, rising from 5.8 years to 6.2 years.

Numerous recent studies have examined the relationship between income and mortality, building upon the seminal work by Chetty et al. (2016), which uncovered a clear and large correlation between household income and individual life expectancy. Our study documents, for the first time, the correlation between household income and survivors' life expectancy while conducting heterogeneity analyses based on standards of living. Blanpain (2018) reports that in France in 2016, there is a substantial 13-year age gap in life expectancy between the first and last vingtiles of the standard of living distribution for males. For females, the gradient is somewhat lower, at 8 years. We quantify here how these differences in individual life expectancy translate into WD. Figure 6 illustrates that WD decreases linearly with standard of living. Females belonging to the bottom of the standard of living distribution have an expected WD of 11.1 years at age 60, while expected WD is only 9.4 years for females at the top of the distribution. Online Appendix B.6 reveals the increase in the overlap of mortality distributions along the household income distribution, as a results of the gains in terms of modal age at death and the compression of mortality distributions. Male WD also exhibits a negative gradient along the household income distribution. Figure 6 demonstrates that men at the bottom of the income distribution have an expected WD of 5.9 years at age 60, while those at the top of the distribution have only 5.4 years.

The comparative advantage of widows from the bottom of the distribution receiving more survival benefits during their extended time as widows is significantly offset by the economic risks associated with widowhood. By adopting a perspective that allows us to examine differential mortality, Figure 6 also illustrates disparities in the joint survival of couples across the income distribution. The figure reveals that couples in the highest income group can anticipate a joint life expectancy of 23.5 years at age 60, whereas those in the lowest income group can only expect 16.8 years of shared life together. The increased risk of poverty among widows may also stem from greater variability in WD within the lowest income segment of the population. An extended WD amplifies the risk of poverty due to two key factors: *(i)* the higher likelihood of adverse events occurring over an extended time frame, and *(ii)* the additional impact of retirees experiencing a

anticipated small decrease in female WD remains consistent. In terms of magnitude, WD is expected to remain relatively high in 2070, ranging from 7 years (under scenarios of low life expectancy, low age gap, and high dependence) to 13.4 years (in scenarios of high life expectancy, high age gap, and low dependence).

decline in their standard of living over time compared to those still in the labor market.[16] The average WD alone does not provide insights into its dispersion. In Figure 7 we present the proportion of widows whose WD is expected to exceed 15, 20, or 25 years per standard of living vingtile. Among widows in the lowest income vingtile, 28% are expected to have a WD exceeding 15 years, whereas this is the case for only 19% in the highest vingtile. Furthermore, 4.5% of widows in the lowest standard of living vingtile are likely to be widows for more than 25 years, compared to only 2.1% in the highest vingtile.

Discussion

In this paper, we estimate the factors influencing WD within a unified framework. We show that in 2020, the expected WD for females at age 60 is 10.4 years, and it depends on three primary determinants: *(i)* the mortality distributions of both females and males, including their degree of overlap; *(ii)* the age gap between spouses; and *(iii)* the dependence of spousal mortality. Starting from our most precise definition of WD, we sequentially isolate each determinant down to the most basic definition, thus quantifying their role in the measurement. First, we are able to consider the dependence of spousal mortality, a factor not previously accounted for in the literature on either WD (Compton and Pollak, 2021) or the probability of widowhood (Bergeron-Boucher et al., 2022). This dependence reduces WD by 3 years. Had we assumed that mortality distributions of spouses are independent, WD would have been 13.4 years. Furthermore, we expanded upon the work of Goldman and Lord (1983), which focused on specific age gap combinations. This reveals that considering the entire age gap distribution among spouses in the population contributes an additional year to WD, with 0.4 years attributed to dispersion around the mean. If we had assumed the same age for spouses, WD would have been 12.4 years. Next, if we consider that the dispersion of ages at death for females is the same as for men, WD would increase by 0.5 years, yielding 12.9 years. Finally, assuming that the female mortality pattern matches that of males, not only having the same dispersion of ages at death but also the same modal age at death, would result in a WD of 10.9 years (a reduction of 2 years).

[16] Pension benefits do not increase as fast as the average income in the economy because they are indexed to inflation.

Our methodology is grounded in the utilization of simulations based on the Gompertz law and a bivariate Gaussian copula. Specifically, we utilize the INSEE life tables as our primary data source (Compton and Pollak, 2021). Building upon the work of Vaupel et al. (2021), we introduce a temporal dimension to their outsurvival indicator. We calculate the probability of widowhood, unconditional widowhood duration (UWD), and widowhood duration conditional on being widowed (WD) using a Gompertz model transformed into a location-scale mortality density (Basellini et al., 2019). This approach enables us to directly analyze the role of compression and right-shifting of mortality in the WD measure. The flexibility of our simulation approach allows us to generate a population of couples with age gaps drawn from observed population distributions (Bouchet-Valat, 2019) and to correlate death ranks among spouses using a Gaussian copula.

From this methodology, we derive two sets of novel results that shed light on important policy questions. First, using INSEE projected life tables for the period 2020–2070, we demonstrate that WD is expected to slightly decrease over this time frame but will remain high. By 2070, the expected WD is projected to be 9.2 years for females at age 60 and 6.3 years for men. This underscores the importance for policymakers to recognize that even with the convergence of life expectancies between females and males, widows and widowers may still experience vulnerability periods. In particular, our UWD indicator offers a valuable measure of the overall level of widowhood protection required by a population. It estimates the average duration during which a widow or widower may potentially be eligible for social benefits, considering that some individuals may not experience widowhood. Second, our analysis of WD heterogeneity based on standard of living reveals that females at the bottom of the distribution experience a longer expected WD compared to females at the top. Furthermore, the dispersion of WD is greater among individuals with lower standards of living, which amplifies their risk of poverty. Additionally, our findings indicate a dual penalty associated with differential mortality. Low-income females not only have shorter lifespans but also spend a larger proportion of their life expectancy in widowhood, leading to increased experiences of loneliness.

At this stage, it is important to acknowledge that we do not address two limitations that have been discussed in the existing literature. First, we utilize life tables computed for the entire population, without using lifetables specific to marital status. Consequently, we are unable to account for the fact that mortality among married individuals tends to be lower than that of singles, possibly due to the healthier individuals self-selecting into marriage (Goldman, 1993) and to the protective effects of marriage (Sanders and Melenberg, 2016). Second, since our focus is on the evolution of WD over time, most of our measures are based on period mortality rates. While we have observed similar patterns in average WD across birth cohorts, it is important to note that the assumption of stability in age-specific mortality rates, upon which most of our results rely, is highly unlikely. Furthermore, we do not present a comprehensive global decomposition of WD. Instead, we examine the impact of each determinant individually while holding all other parameters constant at their central values. In doing so, we do not account for potential endogeneity among these determinants, such as the documented influence of age gap on mortality (Drefahl, 2010). Furthermore, the likelihood of both the age gap between spouses and the level of spousal mortality dependence remaining constant is improbable across the vingtiles of standard of living (Dabergott, 2022). Finally, it should be noted that our calculations are based on the assumption that couples do not divorce before widowhood and survivors do not remarry after widowhood. Although this aligns with the predominant behavior among individuals aged 60 and over, such cases do exist and may become more prevalent in the future, as repartnership after widowhood may become more common at older ages (Brown et al., 2012). From the perspective of policymakers, it is essential to acknowledge that our approach may overestimate the survivor benefits widows would claim and the duration for which they would receive them.[17] However, from an individual point of view, these calculations are valuable for assessing one's vulnerability in widowhood.

Interestingly, under the same hypotheses as Compton and Pollak (2021), our results suggest that, compared to the United States (US), WD in France is slightly lower in 2020, though still within the same range. This difference arises because WD in France has stabilized since the 1990s, while WD in the US exhibits a U-shaped pattern over time. This last observation highlights the need for a more systematic international comparison, which we consider as a potential avenue for future research.

[17] Survivor benefits vary by country and are often reduced or even eliminated in the event of a divorce before widowhood. Furthermore, they may be eliminated in cases of remarriage after widowhood. For details, see OECD (2019).

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Figures

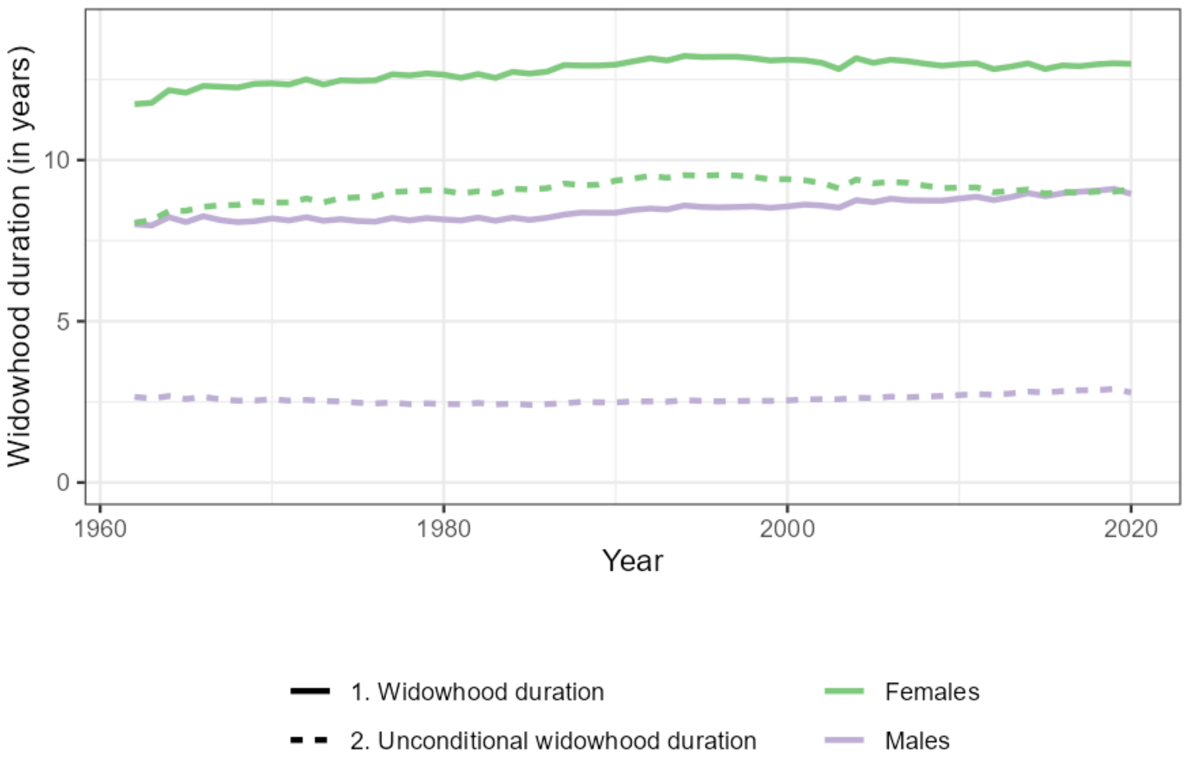


Figure 1: Evolution of widowhood duration duration over the years
Note: We assume a fixed 2-year age gap between spouses.
Source: INSEE life tables.

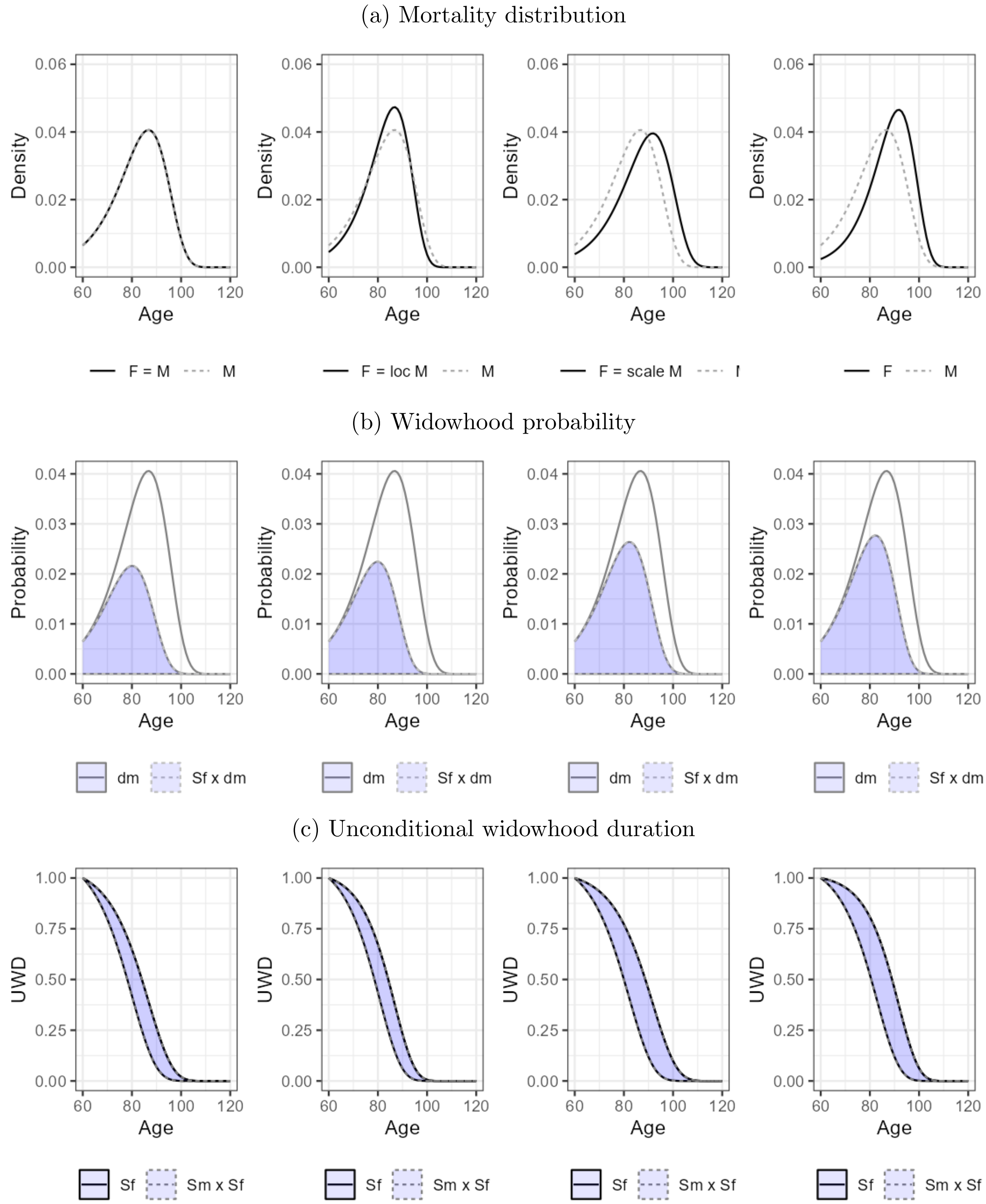


Figure 2: The role of mortality overlap in widowhood duration, in 2020
Note: Mortality distributions are simulated using a Gompertz law and assuming neither an age gap between spouses nor any dependence of spousal mortality. UWD stands for unconditional widowhood duration. M=F stands for the scenario in which women have the same mortality pattern as men. F=loc M stands for the scenario in which we assume that only the location parameter of the Gompertz law for women is the same as that for men. F=scale M stands for the scenario in which we assume that only the scale parameter of the Gompertz law for women is the same as that for men.
Source: Authors' simulations and INSEE life tables.

(a) Age gap between spouses (b) Dependence of spousal mortality

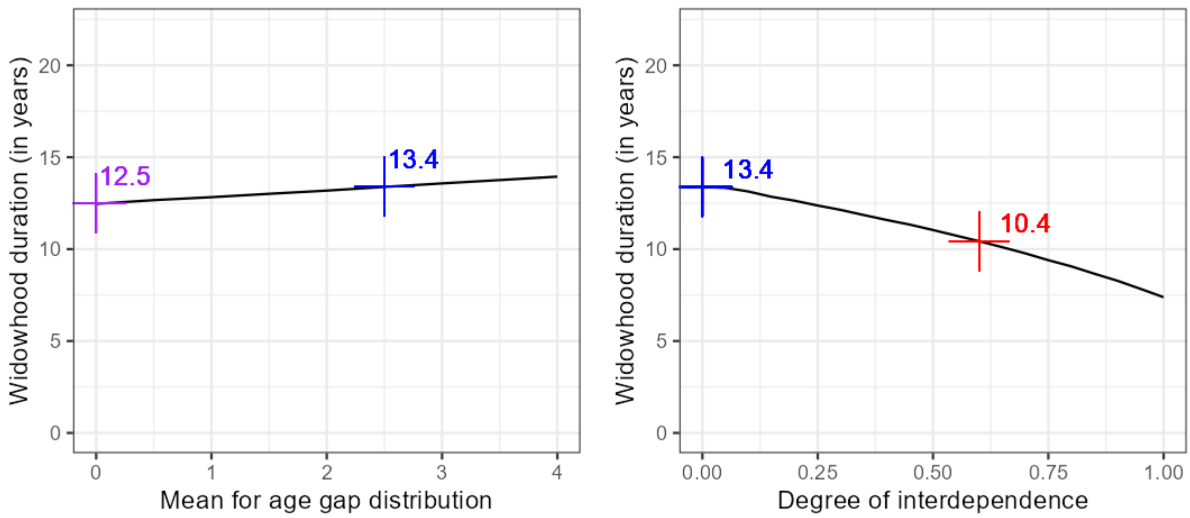


Figure 3: Widowhood duration in relation to age gap and dependence of spousal mortality
Note: Mortality distributions are simulated using a Gompertz law.
Source: Authors' simulations and INSEE life tables.

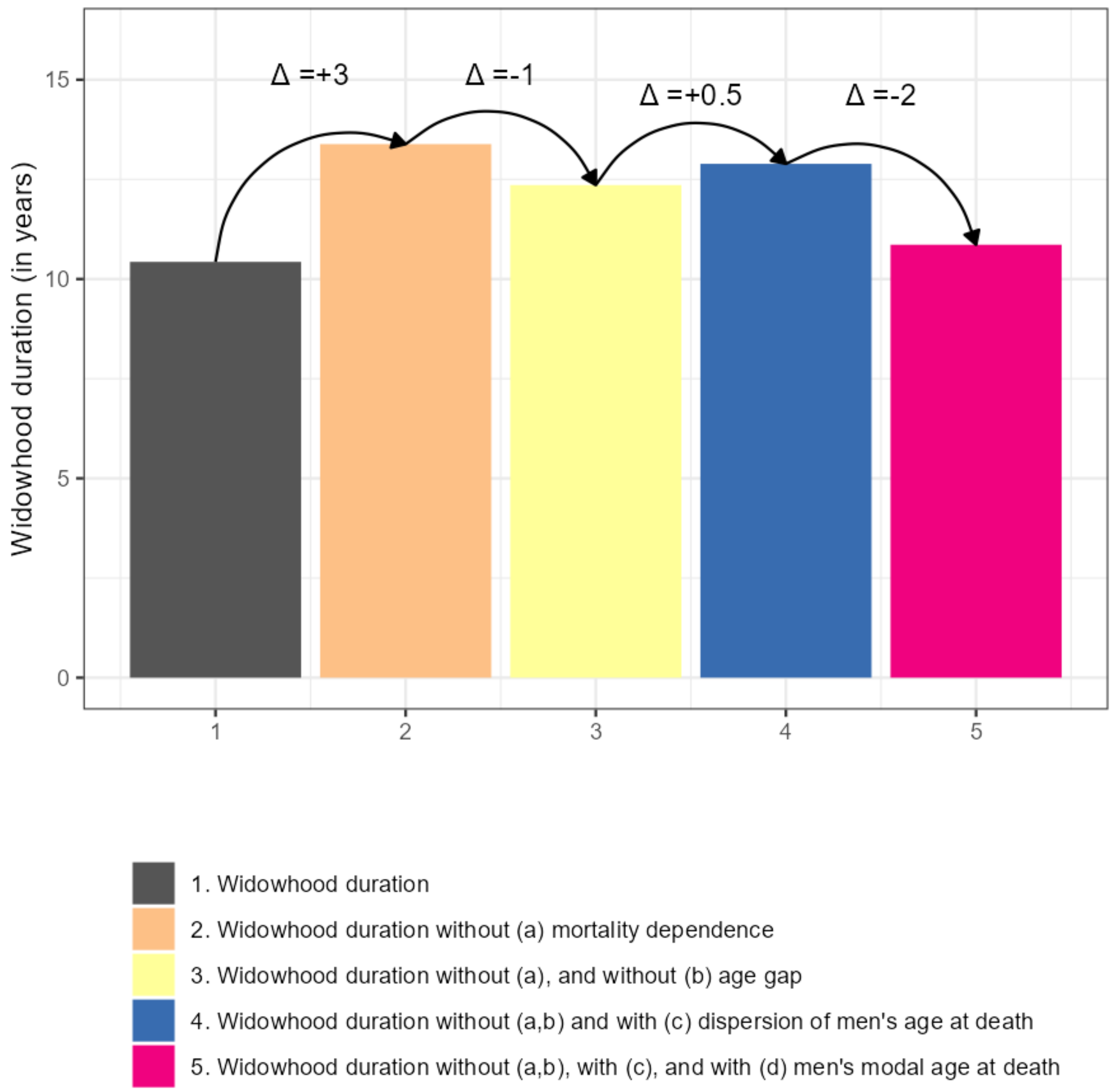


Figure 4: Determinants of female widowhood duration in 2020
Note: In 2020, expected WD at age 60 is 10.4 years. Ignoring the dependence of spousal mortality would have increased WD by 3 years (Δ), to 13.4. Disregarding the existence of an age gap between spouses would have reduced it by 1 year, to 12.4. Considering equal dispersions of ages at death between females and males would have increased WD by 0.5 years, to 12.9. Finally, assuming that the female mortality pattern matches that of males (same modal and dispersion of ages at death) would result in a WD of 10.9 years (2 years lower). Mortality distributions are simulated with a Gompertz law.
Source: Authors' simulations and INSEE life tables.

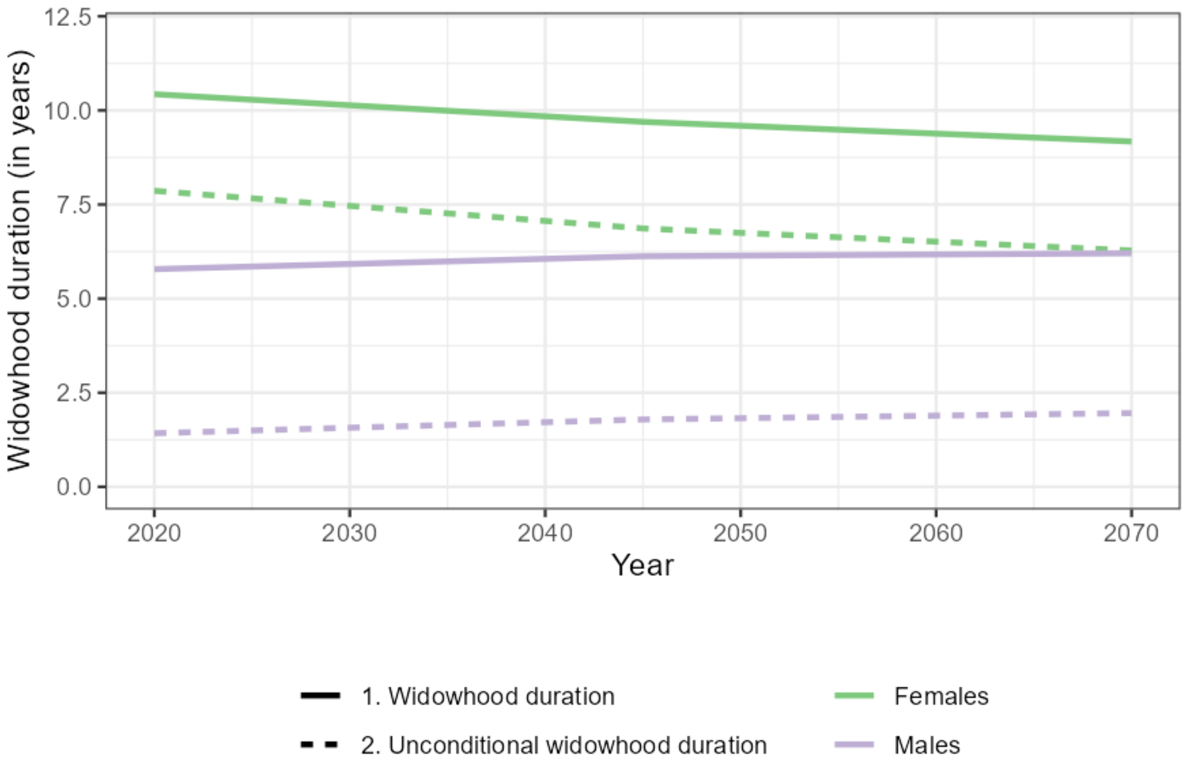


Figure 5: Projected evolution of widowhood duration over the years
Note: Mortality distributions are simulated using a Gompertz law.
Source: Authors' simulations and INSEE life tables.

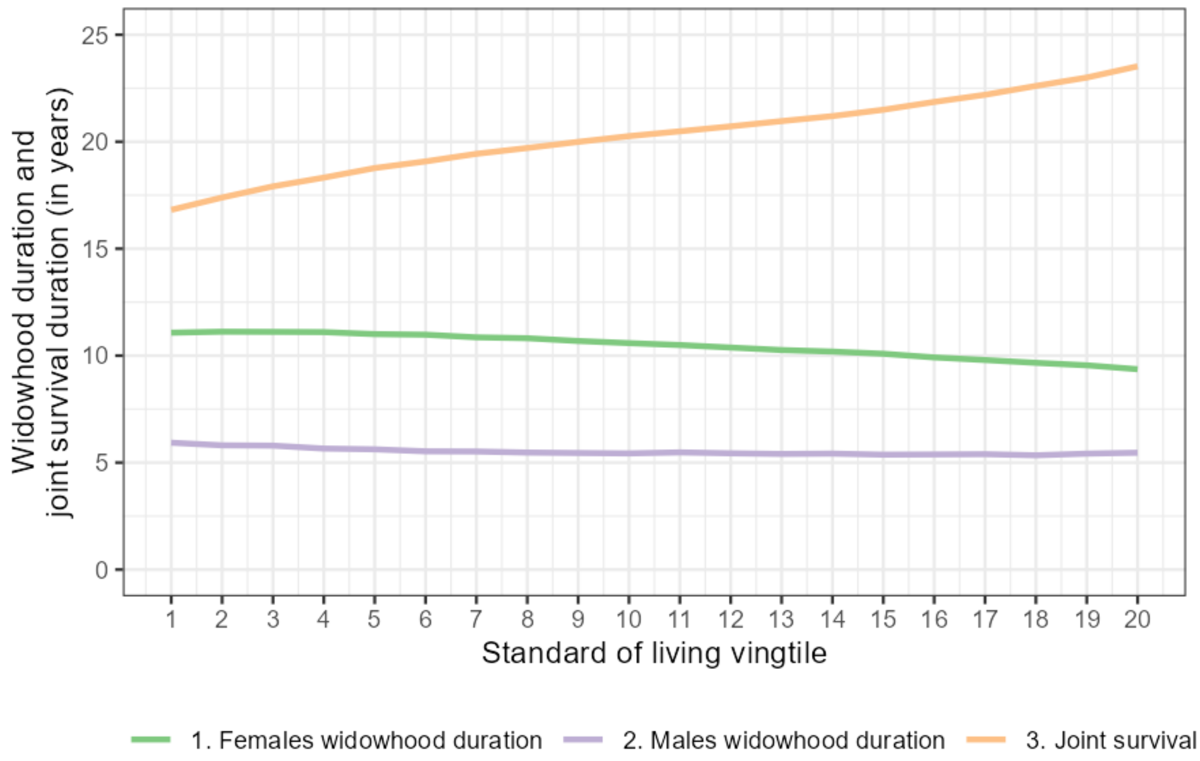


Figure 6: Widowhood and joint survival durations per standard of living in 2016
Note: Mortality distributions are simulated using a Gompertz law.
Source: Authors' simulations and INSEE life tables.

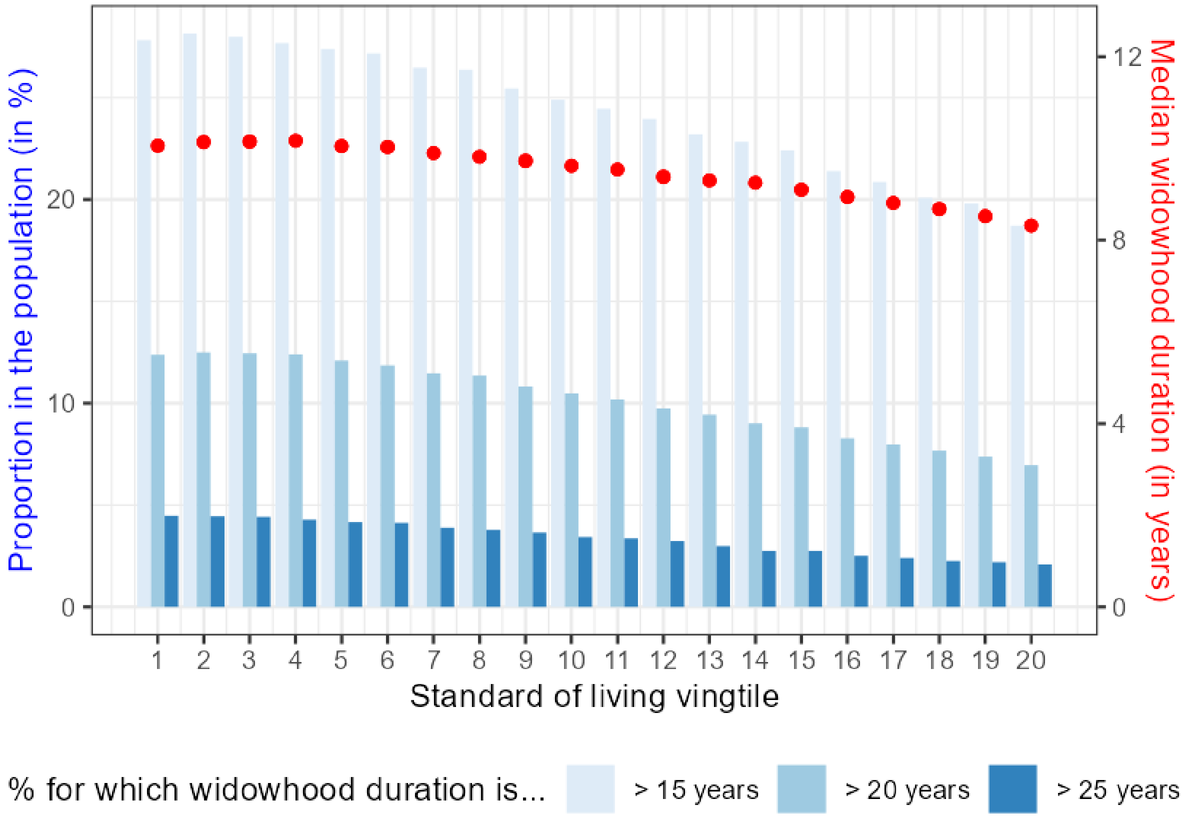


Figure 7: Median and extreme widowhood duration by standard of living (females)
Note: Mortality distributions are simulated using a Gompertz law.
Source: Authors' simulations and INSEE life tables.

A Theoretical complements

A.1 WD without any age gap between spouses

To illustrate why the duration is not equal to the difference between the average life expectancy of men and women, we begin with Equation 5 from the main text:

$$WD_f(x) = \int_x^\omega s_f(u).[1 - s_m(u)].du \tag{A.1}$$

This formula can be expressed differently, as shown in Equation (A.2):

$$WD_f(x) = e_f(x) - \int_x^\omega s_f(u).s_m(u).du = e_f(x) - e_m(x)\frac{\int_x^\omega s_f(u).s_m(u).du}{\int_x^\omega s_m(u).du} \tag{A.2}$$

From Equation (A.2) we observe that $WD_f(x)$ is greater than the difference between individual life expectancies $e_f(x) - e_m(x)$ because $\frac{\int_x^\omega s_f(u).s_m(u).du}{\int_x^\omega s_m(u).du}$ is lower than one.
For this difference $e_f(x) - e_m(x)$ to be equal to $WD_f(x)$, it would require that $s_f(u) = 1$ on all $s_m(u)$ support. In other words, no woman would die before all the men have died.
Conversely, in the scenario where $s_m(u) = 1$ on all $s_f(u)$ support, we obtain:

$$e_f(x) - \int_x^\omega s_f(u).du = 0 \tag{A.3}$$

In this case, there would be no female widowhood, resulting in a duration of zero, whereas the difference between individual life expectancies would yield a negative duration.
Symmetrically, we have:

$$WD_m(x) = e_m(x) - \int_x^\omega s_f(u).s_m(u).du \tag{A.4}$$

which implies equality between the difference in individual life expectancies $e_f(x) - e_m(x)$ and the difference in WD $WD_f(x) - WD_m(x)$.
Calculating WD as the simple difference between the spouses' individual life expectancies would result in negative values when the order of deaths is reversed. This would mean subtracting men's WD from that of women.

A.2 Taking into account the age gap between spouses

We now delve into the question of age differences between spouses. In this section, we assume that males are paired with younger females by a fixed age difference equal to Δ. Formally, the WD formula that incorporates the age gap between spouses is presented as:

$$WD_f(x,\Delta) = \int_x^{\omega} d_m(u+\Delta).s_f(u).e_f(u).du. \tag{A.5}$$

Without repeating the entire calculation, we proceed directly from Equation (5). Additionally, we consider deaths before age x, which is the age at which we observe the woman. Specifically, we examine the cases in which she may already be a widow at that age, implying she will spend all her remaining years of life as a widow.

$$WD_f(x,\Delta) = \frac{1}{s_f(x)} \int_x^{\omega} s_f(u).[1 - s_m(u+\Delta)].du. \tag{A.6}$$

Equation (A.6) compiles the remaining years of life for a woman at age $u > x$, conditional on the probability $s_f(x)$ that she is alive at age x. These years are weighted by the probabilities that her spouse is no longer alive, as determined by the male survival rate $s_m(u+\Delta)$. In this new formula, a woman may become widowed as early as at age x, with a probability of $1 - s_m(x+\Delta)$.

To gain insight into how WD depends on age difference between spouses compared to the difference in life expectancy between spouses, we rewrite Equation (A.6) to reveal a linear combination of these quantities, expressed as $e_f(x) - \alpha_1.e_m(x) + \alpha_2.\Delta$:

$$WD_f(x,\Delta) = \frac{1}{s_f(x)} \int_x^{\omega} s_f(u).[1 - s_m(u) + s_m(u) - s_m(u+\Delta)].du, \tag{A.7}$$

This leads to:

$$WD_f(x,\Delta) = e_f(x) - e_m(x)\frac{\int_x^{\omega} \frac{s_f(u)}{s_f(x)}.s_m(u).du}{\int_x^{\omega} s_m(u)du} + \Delta \int_x^{\omega} \frac{s_f(u)}{s_f(x)} \left[\frac{s_m(u) - s_m(u+\Delta)}{\Delta}\right].du. \tag{A.8}$$

In this latter expression, the first two terms correspond to those in Equation (A.2), with $\frac{\int_x^{\omega} \frac{s_f(u)}{s_f(x)}.s_m(u).du}{\int_x^{\omega} s_m(u).du}$ being less than one. The fact that the term $e_m(x)$ is assigned a coefficient less than one reflects the downward bias associated with calculating WD as a simple difference in individual life expectancies. For the last term, we have:

$$\int_x^\omega \frac{s_f(u)}{s_f(x)} \left[\frac{s_m(u) - s_m(u+\Delta)}{\Delta} \right] .du \approx \int_x^\omega \frac{s_f(u)}{s_f(x)} d_m(u).du \leq \int_x^\omega d_m(u).du = s_m(x) \leq 1. \tag{A.9}$$

Equality to one is only valid if $s_m(x)$ is one and if we have $\frac{s_f(u)}{s_f(x)} = 1$ over the entire $d_m(u)$ support. This represents the extreme hypothesis of an absence of female mortality before all men have died, resulting in a total absence of male widowhood. In the latter formula, multiplying Δ by a coefficient lower than one acknowledges that the age difference between spouses will not prevent some wives from dying less than Δ years after their husbands, nor will it prevent others from dying first. Since Δ intervenes positively, ignoring it (even with a coefficient less than one) introduces an inverse bias of overestimation. Considering that a difference of Δ adds Δ years of widowhood to all women, it disregards the fact that this difference will not prevent some women from dying first and others from dying within less than Δ years after their husbands. This, in turn introduces an upward bias.

B Additional results

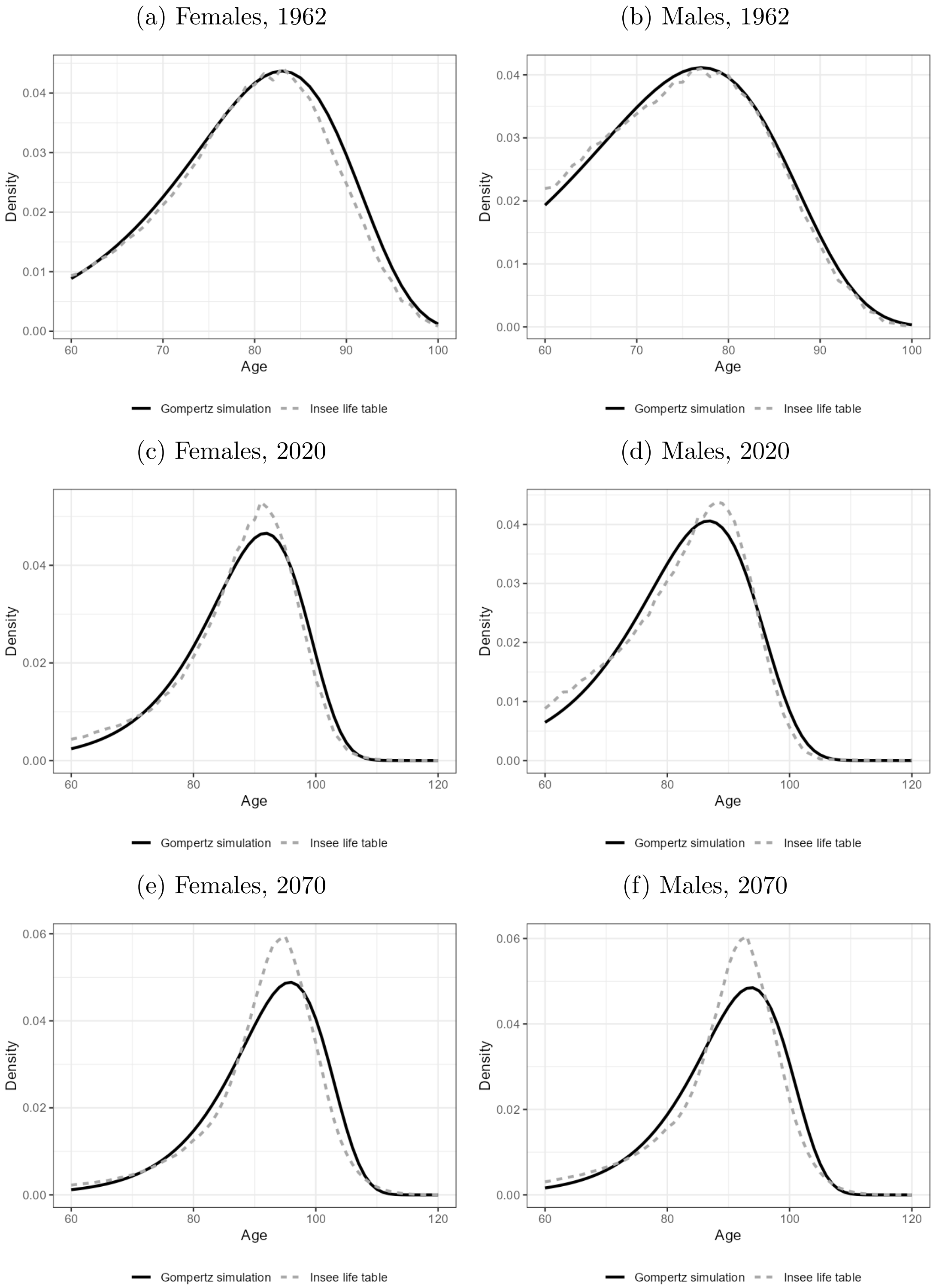


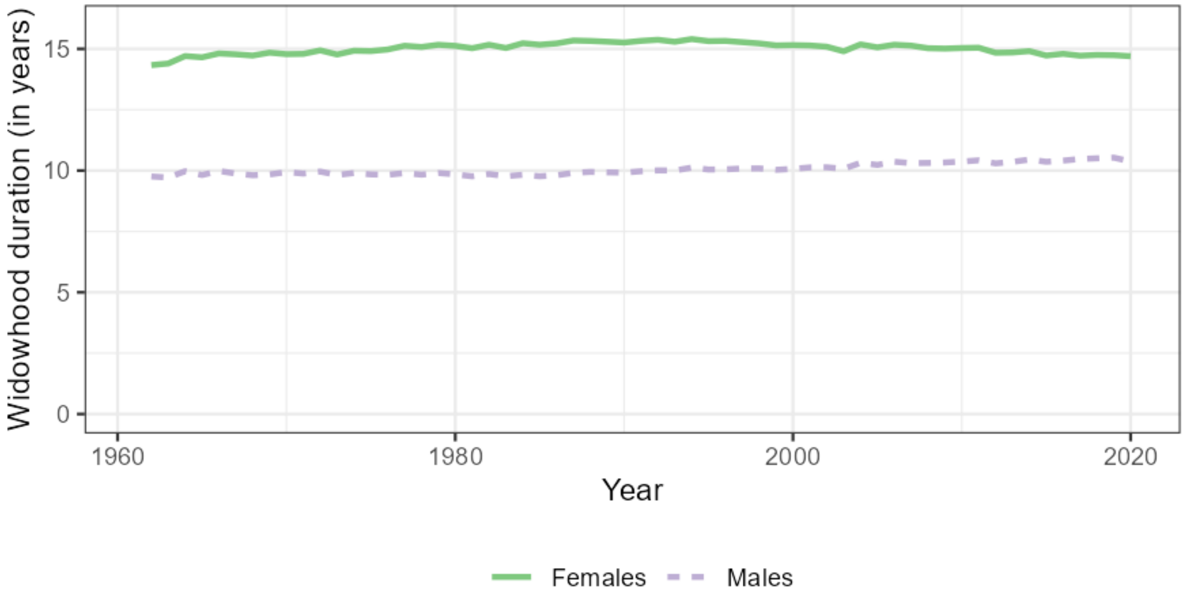
Figure B.1: Goodness of fit
Note: Mortality distributions are simulated using a Gompertz law.
Source: INSEE life tables.

Table B.1: Mortality distribution parameters

	Year			Standard of living vingtile (2016)																			
Parameter	1962	2020	2070	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Location, females	83,0	91,7	95,8	89,0	89,8	90,4	90,8	91,2	91,5	91,7	91,9	92,1	92,2	92,4	92,5	92,6	92,8	92,9	93,1	93,3	93,5	93,7	94,0
Scale, females	9,1	8,1	7,6	9,3	8,9	8,6	8,3	8,1	7,9	7,7	7,6	7,5	7,4	7,3	7,2	7,2	7,1	7,0	6,9	6,9	6,8	6,7	6,6
Location, males	77,1	86,8	93,7	81,0	82,2	83,2	84,0	84,6	85,2	85,7	86,1	86,5	86,9	87,2	87,5	87,9	88,2	88,5	88,9	89,3	89,8	90,3	90,9
Scale, males	11,1	9,6	7,7	12,9	12,3	11,7	11,2	10,8	10,5	10,2	10,0	9,7	9,5	9,2	9,0	8,8	8,5	8,3	8,1	8,0	7,8	7,7	7,6

Note: Parameters are estimated using a Gompertz law, with two parameters (location and scale), for females and males separately.
Source: INSEE life tables.

(a) Expected widowhood duration at age 50



(b) Evolution of widowhood duration over the birth cohorts

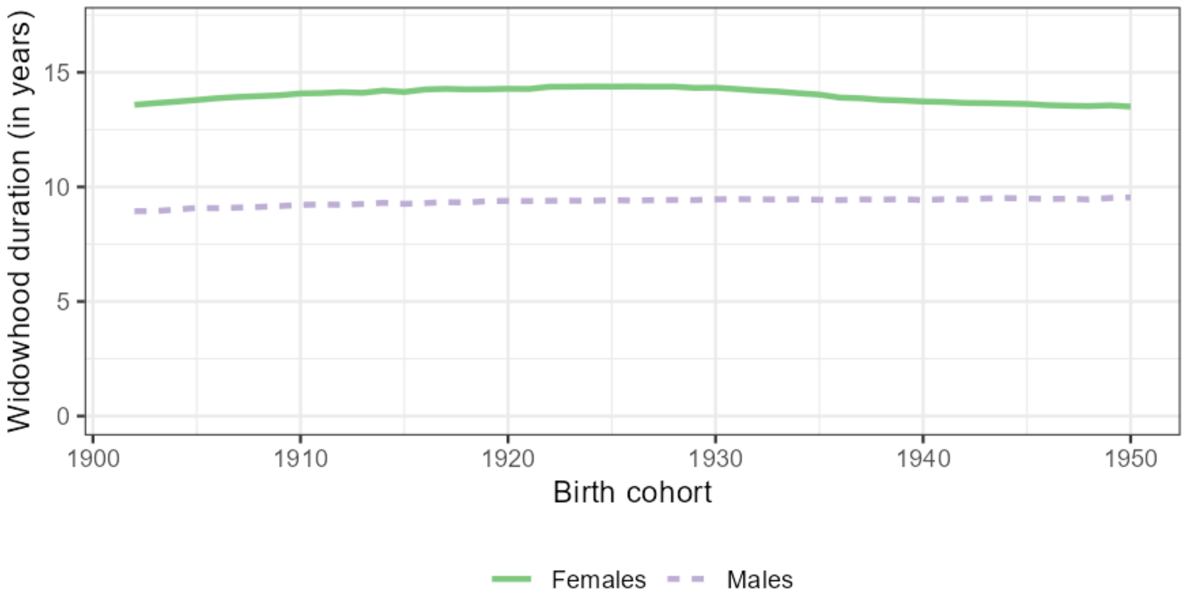


Figure B.2: Expected widowhood duration at age 50 and by birth cohort
Source: INSEE life tables.

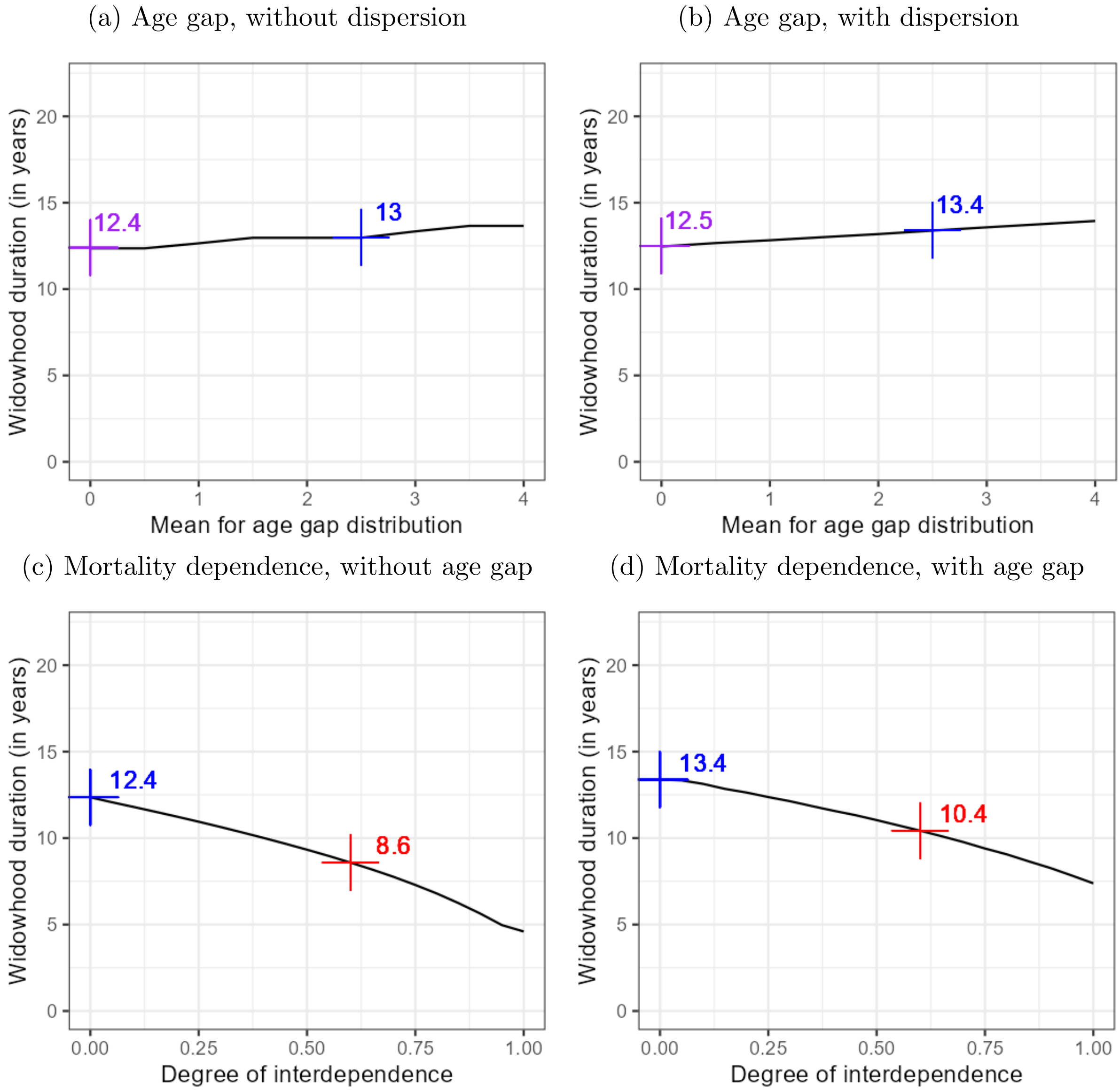


Figure B.3: The role of the age gap and the dependence of spousal's mortality in widowhood duration
Note: Mortality distributions are simulated using a Gompertz law.
Source: Authors' simulations and INSEE life tables.

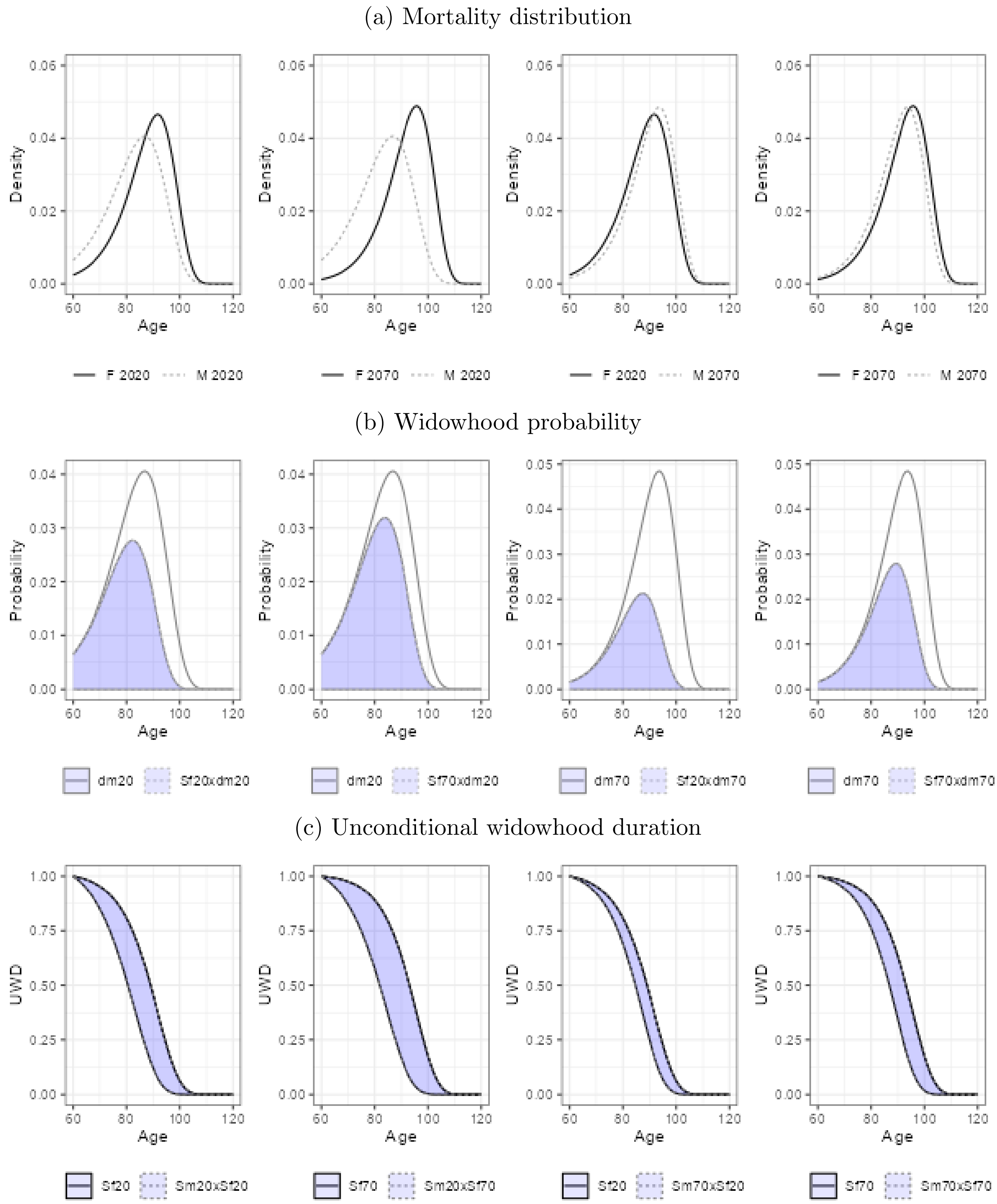
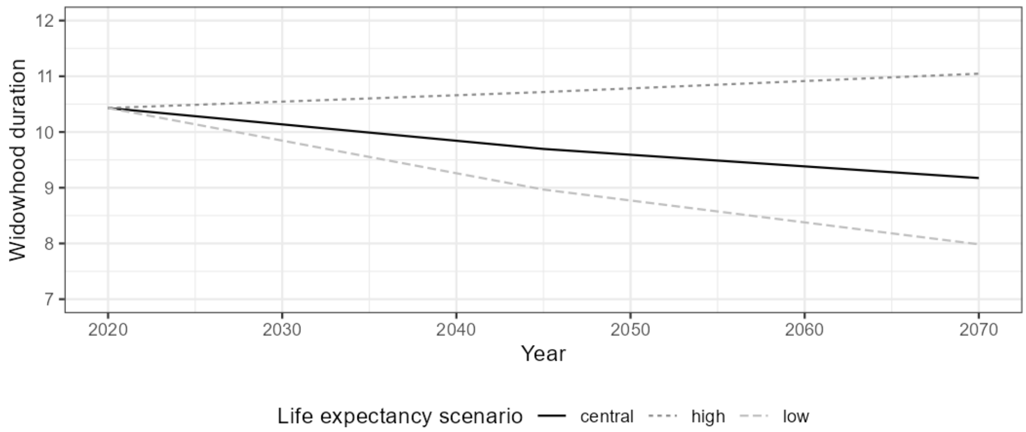
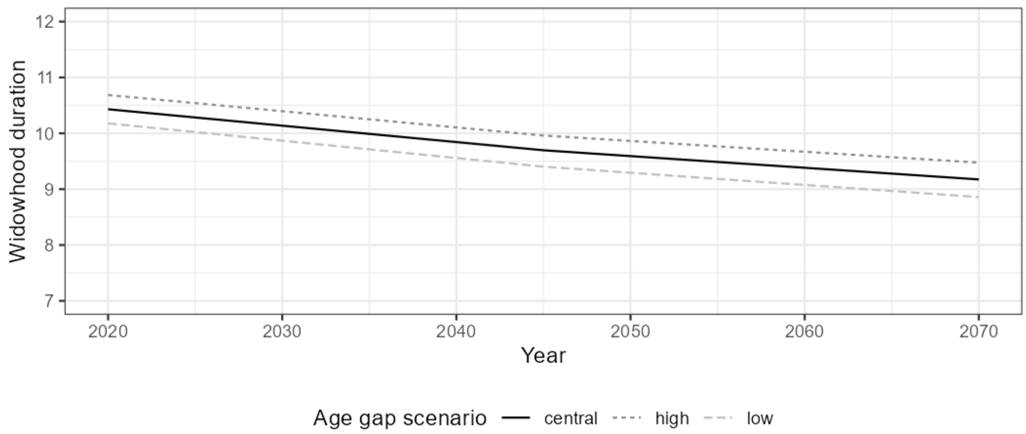


Figure B.4: The role of overlapping mortalities in widowhood duration (2020–2070)
Note: Mortality distributions are simulated using a Gompertz law. Our simulations do not consider an age gap between spouses nor any dependence of spousal mortality. UWD stands for unconditional widowhood duration.
Source: Authors' simulations and INSEE life tables.

(a) Robustness to life expectancy scenario



(b) Robustness to age gap scenario



(c) Robustness to dependence of spousal mortality scenario

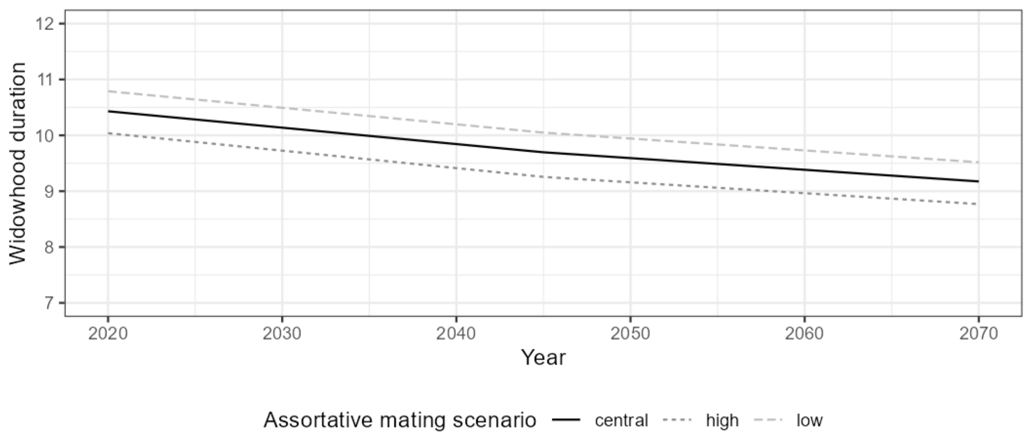


Figure B.5: Robustness to mortality, age gap and degree of dependence scenarios
Note: Mortality distributions follow a Gompertz law. Life expectancy scenarios include INSEE's central, low, and high scenarios (Algava and Blanpain, 2021). Spousal age gap is modeled as a Normal distribution with a mean of 2.5 years and 5-year standard deviation in the central scenario (Bouchet-Valat, 2019). Low and high scenarios adjust the Normal law's parameters by 10% lower and higher, respectively. Dependence of spousal mortality is represented by a Gaussian copula with a parameter of 0.6 (Kaishev et al., 2007). Low and high scenarios adjust copula parameters by 10% lower and higher, respectively.
Source: Authors' simulations and INSEE life tables.

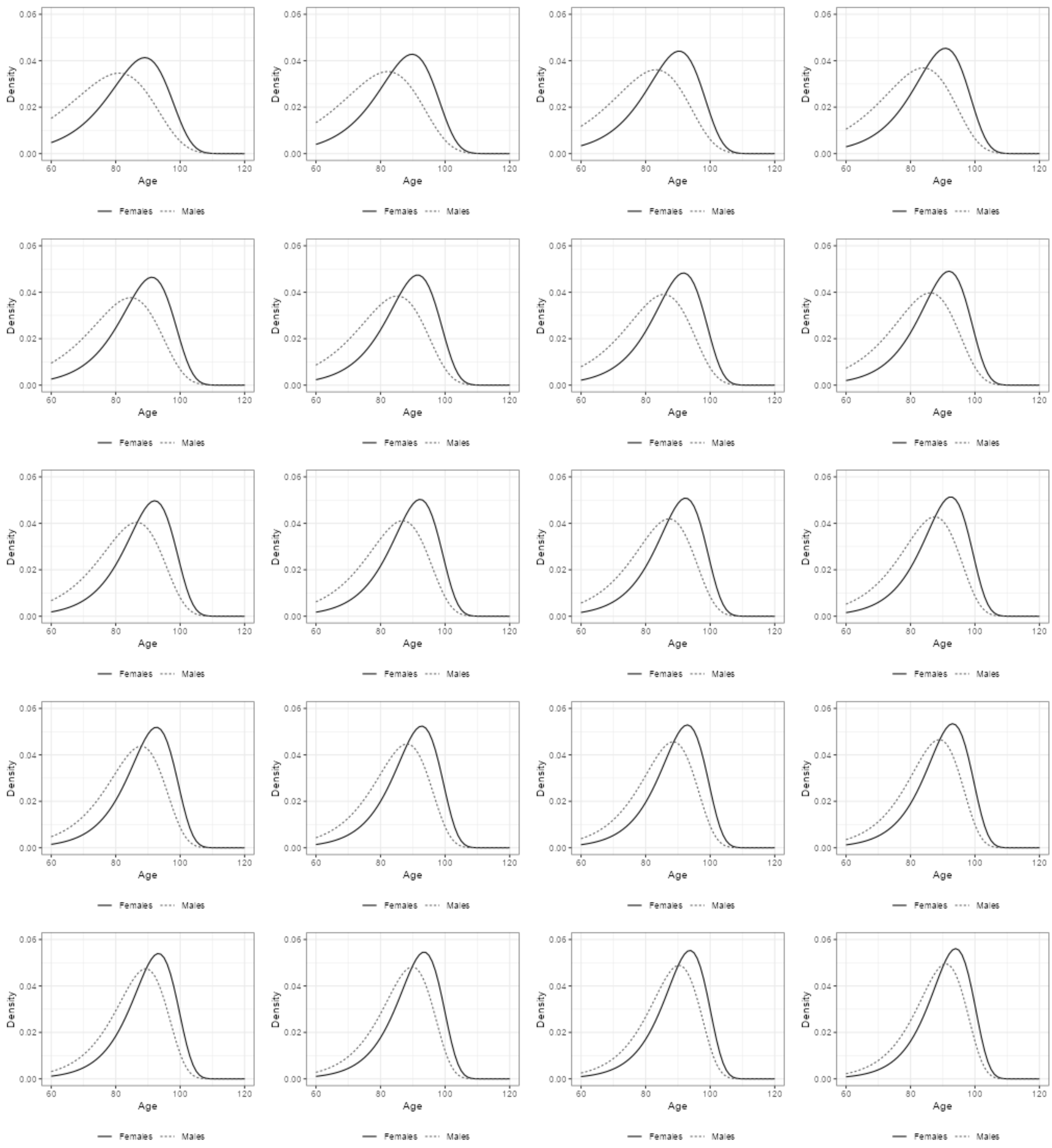


Figure B.6: Overlap of mortality distributions by standard of living
Note: Mortality distributions are simulated using a Gompertz law.
Source: Authors' simulations and INSEE life tables.

C Robustness to computational method

In this paper, we utilize three distinct computation techniques: *(i)* life tables following the methodology of Myers (1959), *(ii)* area under the curve analysis, and *(iii)* simulated population modeling. We employ two types of datasets: *(a)* INSEE life tables and *(b)* Gompertz law data. Table C.1 provides a comprehensive overview of how the metric responds to different data sources, providing insights into the robustness of the WD measurement. We also acknowledge the potential sources of bias that may arise from the utilization of various computational methods.

Table C.1: Robustness of widowhood duration to computational method

Source	Technique	Widowhood duration	In the paper
INSEE life table	Myers (1959)	12.48	Graph 1
INSEE life table	Area under the curve	12.31	-
Gompertz law	Area under the curve	12.35	Table 4
Gompertz law	simulated population	12.38	Graph B.3a

Note: Widowhood duration in 2020, with no age gap between spouses or dependence of spouses' mortalities.
Source: INSEE life tables.

Moving from discrete to continuous formula. To facilitate understanding, we simplify the WD measure by representing it as the area between female and joint survival curves. This approach differs from the computation method presented by Myers (1959), as the latter relies on a discrete time definition, while the former adopts a continuous approach. The discrete time formula is essentially the sum over ages of the area of a rectangle with a width equal to 1 and a height equal to the survival value for the given age. Both formulas yield identical results when the survival curve shows a nearly linear progression with a slope of approximately -1.

Goodness of fit. To enhance clarity regarding the role of determinants in WD, we employ a Gompertz law model with adjustable parameters. Utilizing INSEE life tables as our foundation, we estimate the primary parameters using the 'phreg' function in R (from the 'eha' package) with the default estimation method option, 'efron' (a method for handling ties). Discrepancies between the observed and estimated mortality distributions may result in variations between the two WD measures.

Population modeling. For the sake of realism, we employ a simulated population that incorporates age gaps and considers the dependence of spousal mortality. This approach, involving a substantial number of individuals in our simulated population, helps mitigate potential measurement bias.